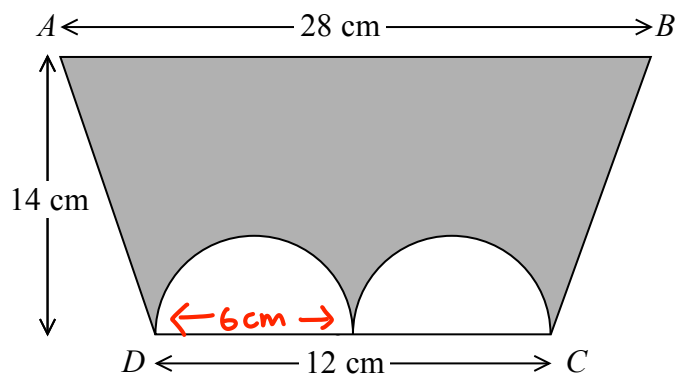




- 1 The diagram shows a trapezium $ABCD$ and two identical semicircles.



The centre of each semicircle is on DC .

Work out the area of the shaded region.

Give your answer correct to 3 significant figures.

$$\text{Trapezium area} = \frac{1}{2}(a+b)h$$

$$\frac{1}{2}(12+28) \times 14 = 280 \text{ cm}^2$$

$$\pi \times 3^2 = 9\pi$$

$$280 - 9\pi = 251.72 \dots$$

$$252 \text{ cm}^2$$

(Total for Question 1 is 4 marks)



- 2 Asif is going on holiday to Turkey.

The exchange rate is £1 = 3.5601 lira.

Asif changes £550 to lira.

- (a) Work out how many lira he should get.
Give your answer to the nearest lira.

$$550 \times 3.5601 \\ = 1958.055$$

1958 lira
(2)

Asif sees a pair of shoes in Turkey.
The shoes cost 210 lira.

Asif does not have a calculator.

He uses £2 = 7 lira to work out the approximate cost of the shoes in pounds.

- (b) Use £2 = 7 lira to show that the approximate cost of the shoes is £60

$$\begin{array}{l} \text{£}2 = 7\text{L} \\ \times 30 \quad \downarrow \quad \text{£}60 = 210\text{L} \quad \downarrow \quad \times 30 \end{array}$$

£1 = 3.5 lira (2)

- (c) Is using £2 = 7 lira instead of using £1 = 3.5601 lira a sensible start to Asif's method to work out the cost of the shoes in pounds?

You must give a reason for your answer.

Yes as £1 = 3.5 lira is a good approximation and easy to calculate without a calculator

(1)

(Total for Question 2 is 5 marks)

- 3 Here are the first six terms of a Fibonacci sequence.

1 1 2 3 5 8 13 21 34

The rule to continue a Fibonacci sequence is,

the next term in the sequence is the sum of the two previous terms.

- (a) Find the 9th term of this sequence.

34

(1)

The first three terms of a different Fibonacci sequence are

a b $a + b$

- (b) Show that the 6th term of this sequence is $3a + 5b$

$$4^{\text{th}} : b + a + b = a + 2b$$

$$5^{\text{th}} : a + b + a + 2b = 2a + 3b$$

$$6^{\text{th}} : a + 2b + 2a + 3b = 3a + 5b$$

(2)

Given that the 3rd term is 7 and the 6th term is 29,

- (c) find the value of a and the value of b .

$$\begin{array}{r} \times 3 \quad \times 3 \quad \times 3 \\ a + b = 7 \end{array}$$

$$3a + 5b = 29$$

$$\begin{array}{r} 3a + 5b = 29 \\ - \quad 3a + 3b = 21 \\ \hline \end{array}$$

$$\begin{array}{r} 2b = 8 \\ b = 4 \end{array}$$

$$\begin{array}{r} a + b = 7 \\ a + 4 = 7 \\ a = 3 \end{array}$$

$$a = 3, b = 4$$

(3)

(Total for Question 3 is 6 marks)

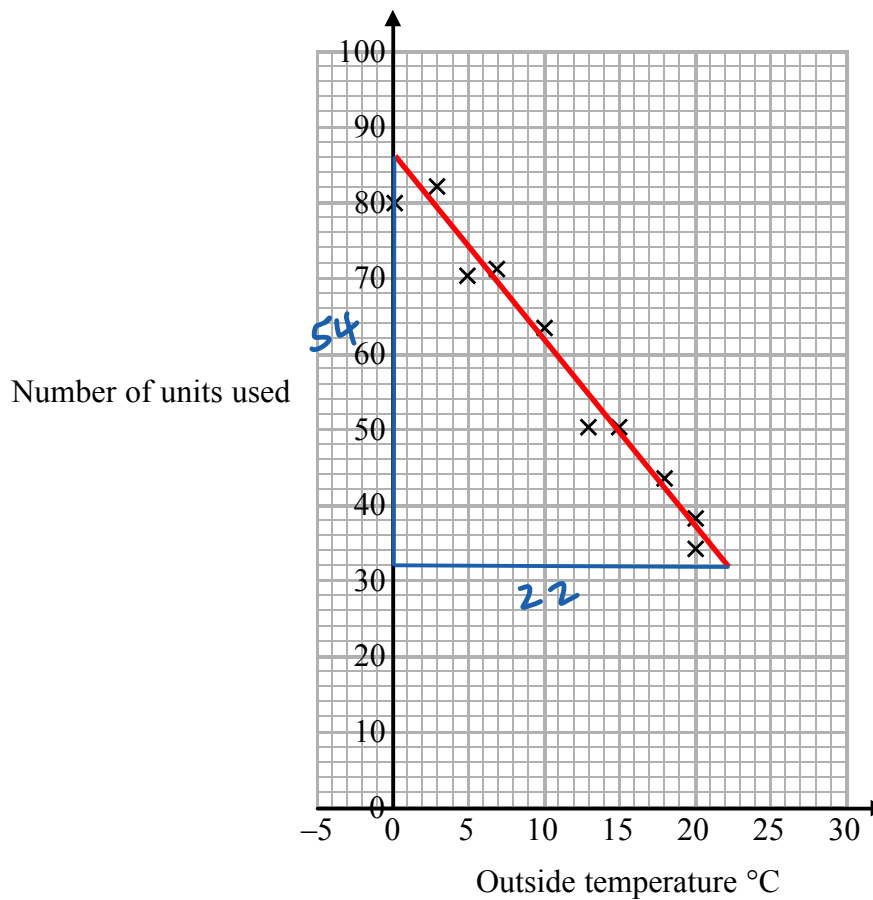
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- 4 In a survey, the outside temperature and the number of units of electricity used for heating were recorded for ten homes.

The scatter diagram shows this information.



Molly says,

“On average the number of units of electricity used for heating decreases by 4 units for each °C increase in outside temperature.”

- (a) Is Molly right?
Show how you get your answer.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

$$\text{Gradient} = -54 \div 22 = -2.45 \quad (-2 \text{ to } -3)$$

No, molly is wrong

(3)

- (b) You should **not** use a line of best fit to predict the number of units of electricity used for heating when the outside temperature is 30 °C.

Give one reason why.

30°C is outside the range of data

(1)

(Total for Question 4 is 4 marks)

5 Henry is thinking of having a water meter.

These are the two ways he can pay for the water he uses.

Water Meter

A charge of £28.20 per year
plus 365 days

91.22p for every cubic metre of water used

1 cubic metre = 1000 litres

No Water Meter

A charge of £107 per year

Henry uses an average of 180 litres of water each day.

Use this information to determine whether or not Henry should have a water meter.

Water meter

$$\begin{aligned}180 \times 365 &= 65700 \text{ L per year} \\ \div 1000 &= 65.7 \text{ cubic metre} \\ \times 91.22 \text{ p} &= 5993.15 \text{ p} \\ &= £59.93 \\ £59.93 + £28.20 &= £88.13\end{aligned}$$

Yes, Henry Should get a water meter

(Total for Question 5 is 5 marks)



6 Liz buys packets of coloured buttons.

There are 8 red buttons in each packet of red buttons.

There are 6 silver buttons in each packet of silver buttons.

There are 5 gold buttons in each packet of gold buttons.

Liz buys equal numbers of red buttons, silver buttons and gold buttons.

How many packets of each colour of buttons did Liz buy?

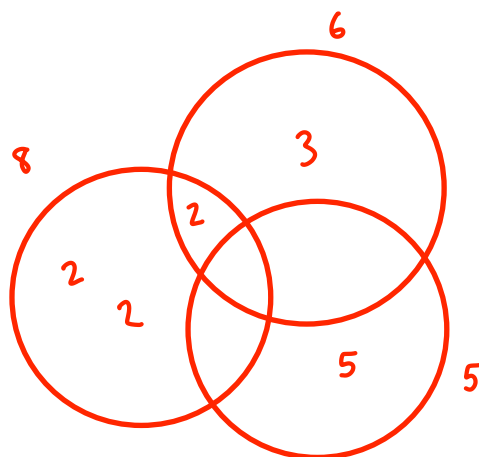
LCM of 8, 6 and 5

$$8 = 2 \times 2 \times 2$$

$$6 = 2 \times 3$$

$$5 = 5$$

$$\begin{aligned} \text{LCM} &= 2 \times 2 \times 2 \times 3 \times 5 \\ &= 120 \text{ buttons} \end{aligned}$$



15 packets of red buttons

20 packets of silver buttons

24 packets of gold buttons

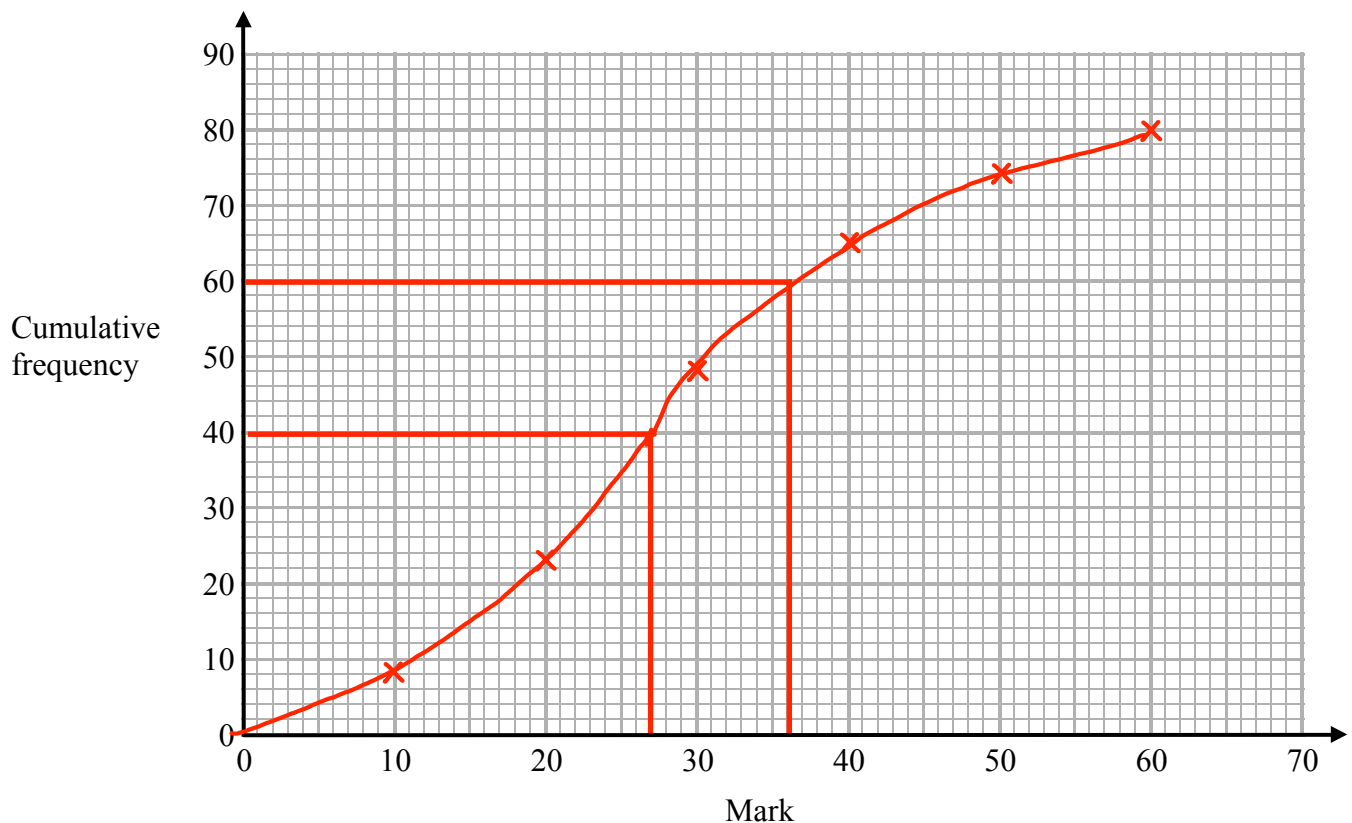
(Total for Question 6 is 3 marks)

7 The cumulative frequency table shows the marks some students got in a test.

Tip: You plot the cumulative frequency at the end point

Mark (m)	Cumulative frequency
$0 < m \leq 10$	8
$0 < m \leq 20$	23
$0 < m \leq 30$	48
$0 < m \leq 40$	65
$0 < m \leq 50$	74
$0 < m \leq 60$	80

(a) On the grid, plot a cumulative frequency graph for this information.



(2)

(b) Find the median mark.

27
(1)
(25 to 28)



Students either pass the test or fail the test.

The pass mark is set so that 3 times as many students fail the test as pass the test.

(c) Find an estimate for the lowest possible pass mark.

75% pass

60 pass

36

(35 to 38)

(Total for Question 7 is 6 marks)



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8 Write 0.000068 in standard form.

$$6.8 \times 10^{-5}$$

(Total for Question 8 is 1 mark)



- 9 (a) Factorise $y^2 + 7y + 6$

$$(y+6)(y+1)$$

(2)

- (b) Solve $6x + 4 > x + 17$

$$\begin{array}{rcl} -x & & -x \\ 5x + 4 & > & 17 \\ -4 & & -4 \\ 5x & > & 13 \\ \div 5 & & \div 5 \\ x & > & 2.6 \end{array}$$

$$x > 2.6$$

(2)

- (c) n is an integer with $-5 < 2n \leq 6$

Write down all the values of n

$$\begin{array}{l} -5 < 2n \leq 6 \\ \div 2 \\ -2.5 < n \leq 3 \end{array}$$

$$-2, -1, 0, 1, 2, 3$$

(2)

(Total for Question 9 is 6 marks)

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DO NOT WRITE IN THIS AREA



10 The function f is such that

$$f(x) = 4x - 1$$

(a) Find $f^{-1}(x)$

$$\begin{array}{l|l} y & = 4x - 1 \\ +1 & \quad +1 \\ \hline y+1 & = 4x \\ \div 4 & \quad \div \\ \hline \frac{y+1}{4} & = x \end{array}$$

$$f^{-1}(x) = \frac{y+1}{4} \quad (2)$$

The function g is such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant.}$$

Given that $fg(2) = 12$

(b) work out the value of k

$$\begin{aligned} g(2) &= k \times 2^2 \\ &= 4k \end{aligned}$$

$$\begin{aligned} f(4k) &= 4(4k) - 1 \\ &= 16k - 1 \end{aligned}$$

$$16k - 1 = 12$$

$$16k = 13$$

$$k = 13/16$$

$$k = \frac{13}{16} \quad (2)$$

(Total for Question 10 is 4 marks)

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11 Solve $x^2 - 5x + 3 = 0$

Give your solutions correct to 3 significant figures.

$$a = 1$$

$$b = -5$$

$$c = 3$$

$$\frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = 4.30$$

$$x = 0.697$$

(Total for Question 11 is 3 marks)



12 Sami asked 50 people which drinks they liked from tea, coffee and milk.

All 50 people like at least one of the drinks

19 people like all three drinks.

16 people like tea and coffee but do **not** like milk.

21 people like coffee and milk.

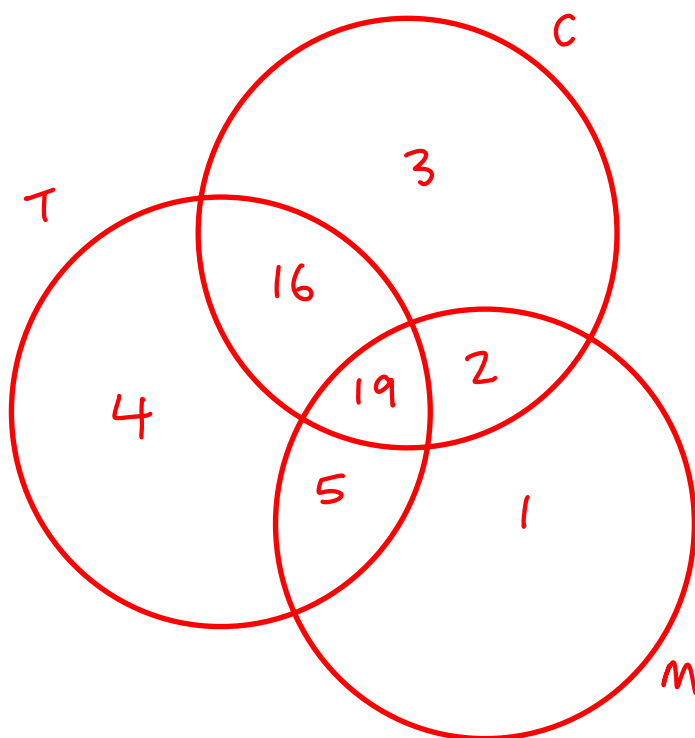
24 people like tea and milk.

40 people like coffee.

1 person likes only milk.

Sami selects at random one of the 50 people.

(a) Work out the probability that this person likes tea.



$$\frac{44}{50}$$

(4)

(b) Given that the person selected at random from the 50 people likes tea, find the probability that this person also likes exactly one other drink.

$$\text{Tea} = 44$$

$$\text{Tea} + \text{milk} = 5$$

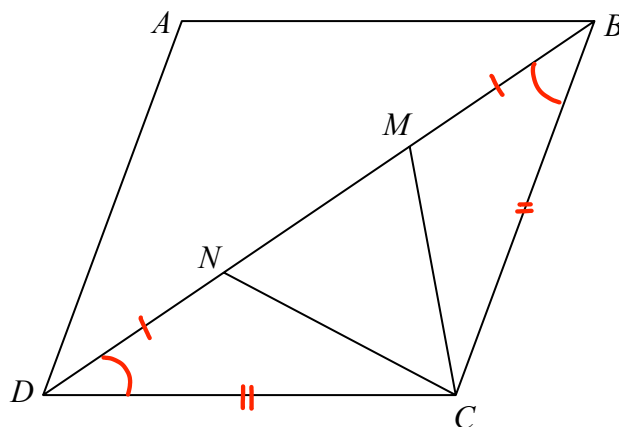
$$\text{Tea} + \text{coffee} = 16$$

$$\frac{21}{44}$$

(2)

(Total for Question 12 is 6 marks)

13 $ABCD$ is a rhombus.



M and N are points on BD such that $DN = MB$.

Prove that triangle DNC is congruent to triangle BMC .

$DN = MB$

$\angle BDC = \angle CBD$ as base angles of an isosceles triangle are equal

$DC = BC$ as all sides of a rhombus are the same

Therefore, congruent as SAS proven

(Total for Question 13 is 3 marks)

- 14 (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

$$x^3 + 4x - 1 = 0$$

when $x = 0$

$$(0)^3 + 4(0) - 1 = -1$$

when $x = 1$

$$(1)^3 + 4(1) - 1 = 4$$

There has been a change in sign $\therefore$ there is a solution

(2)

- (b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

$$\begin{array}{r|l} -x^3 & -x^3 \\ 4x & = 1 - x^3 \\ \div 4 & \div 4 \\ x & = \frac{1}{4} - \frac{x^3}{4} \end{array}$$

(1)

- (c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$ twice, to find an estimate for the solution of $x^3 + 4x = 1$

$$x_1 = \frac{1}{4} - \frac{(0)^3}{4} = 0.25$$

$$x_2 = \frac{1}{4} - \frac{(0.25)^3}{4} = \frac{63}{256}$$

$$\frac{63}{256}$$

(3)

(Total for Question 14 is 6 marks)

- 15 There are 17 men and 26 women in a choir.
The choir is going to sing at a concert.

One of the men and one of the women are going to be chosen to make a pair to sing the first song.

- (a) Work out the number of different pairs that can be chosen.

$$17 \times 26 = 442$$

442

(2)

Two of the men are to be chosen to make a pair to sing the second song.

Ben thinks the number of different pairs that can be chosen is 136

Mark thinks the number of different pairs that can be chosen is 272

- (b) Who is correct, Ben or Mark?
Give a reason for your answer.

$$17 \times 16 = 272 \div 2 = 136$$

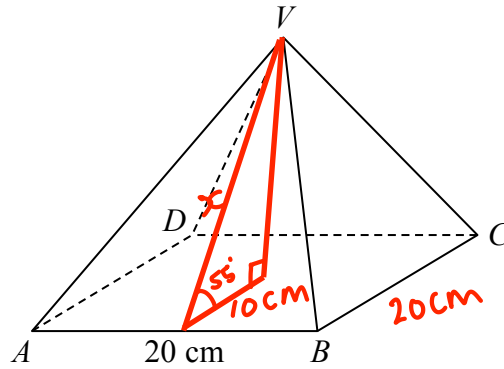
Ben is correct

(1)

(Total for Question 15 is 3 marks)



16 $VABCD$ is a solid pyramid.

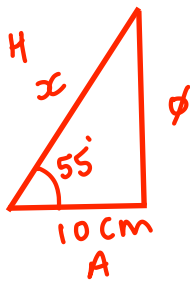


$ABCD$ is a square of side 20 cm.

The angle between any sloping edge and the plane $ABCD$ is 55°

Calculate the surface area of the pyramid.

Give your answer correct to 2 significant figures.



$$\sin 55^\circ = \frac{h}{x} \quad \cos 55^\circ = \frac{10}{x}$$

$$\frac{10}{\cos(55)} = 17.4344...$$

$$\text{Area Square} = 20 \times 20 = 400 \text{ cm}^2$$

$$\text{Area triangle} = \frac{20 \times 17.43}{2} = 174.34 \text{ cm}^2$$

$$\times 4 = 697.38 \text{ cm}^2$$

$$400 + 697.38 = 1097.38 \text{ cm}^2$$

$$1100 \text{ cm}^2$$

(Total for Question 16 is 5 marks)



- 17 Louis and Robert are investigating the growth in the population of a type of bacteria. They have two flasks A and B.

At the start of day 1, there are 1000 bacteria in flask A.

The population of bacteria grows exponentially at the rate of 50% per day.

- (a) Show that the population of bacteria in flask A at the start of each day forms a geometric progression.

$$\begin{aligned}
 \text{day 1} &= 1000 \\
 \text{day 2} &= 1000 \times 1.5 = 1500 \\
 \text{day 3} &= 1500 \times 1.5 = 2250 \\
 &\times 1.5 \text{ each time}
 \end{aligned}
 \tag{2}$$

The population of bacteria in flask A at the start of the 10th day is k times the population of bacteria in flask A at the start of the 6th day.

- (b) Find the value of k .

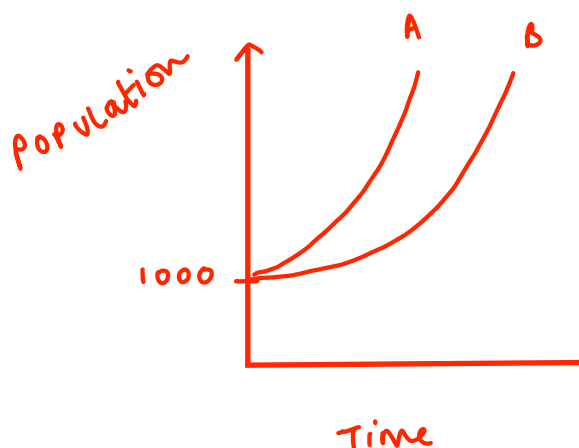
$$\begin{aligned}
 1000 \times 1.5^9 &= K(1000 \times 1.5^5) \\
 \frac{1000 \times 1.5^9}{1000 \times 1.5^5} &= K \\
 5.0625 &= K
 \end{aligned}$$

$\underline{5.0625}$
 (2)

At the start of day 1 there are 1000 bacteria in flask B.

The population of bacteria in flask B grows exponentially at the rate of 30% per day.

- (c) Sketch a graph to compare the size of the population of bacteria in flask A and in flask B.

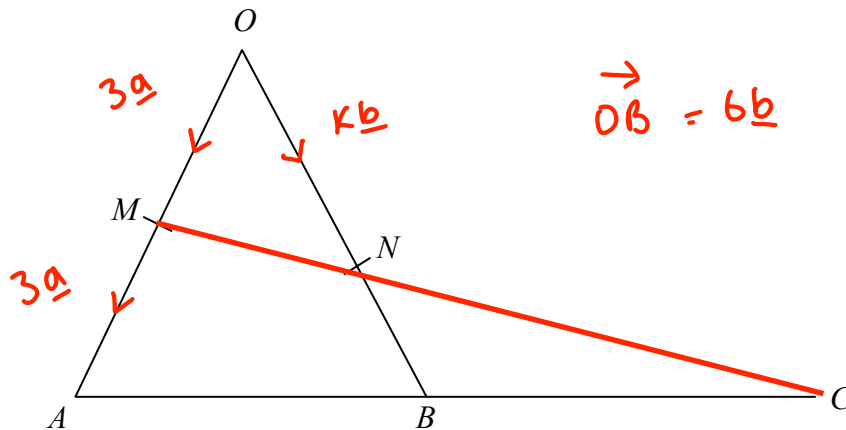


(1)

(Total for Question 17 is 5 marks)



18



OMA , ONB and ABC are straight lines.

M is the midpoint of OA .

B is the midpoint of AC .

$\vec{OA} = 6\mathbf{a}$ $\vec{OB} = 6\mathbf{b}$ $\vec{ON} = k\mathbf{b}$ where k is a scalar quantity.

Given that MNC is a straight line, find the value of k .

$$\vec{MN} = -3\mathbf{a} + k\mathbf{b}$$

$$\vec{AB} = -6\mathbf{a} + 6\mathbf{b}$$

$$\vec{AC} = -12\mathbf{a} + 12\mathbf{b}$$

$$\vec{MC} = 3\mathbf{a} - 12\mathbf{a} + 12\mathbf{b} = -9\mathbf{a} + 12\mathbf{b}$$

$\vec{MC}$ is a multiple of $\vec{MN}$

$$\vec{MN} = -3\mathbf{a} + k\mathbf{b}$$

$$\vec{MC} = -9\mathbf{a} + 12\mathbf{b} \quad \left. \begin{array}{l} \vec{MN} \\ \vec{MC} \end{array} \right\} \times 3$$

$$3k = 12$$

$$k = 4$$

(Total for Question 18 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS